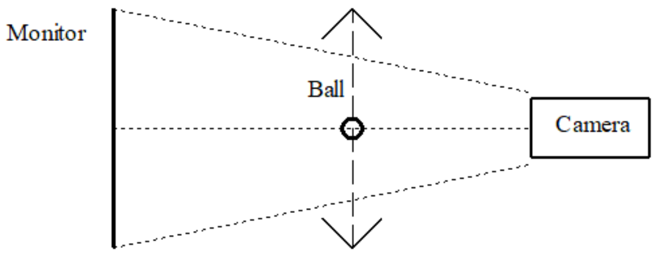
CAMERA INCEPTION: THE DROSTE EFFECT

If you point a camera back at its own monitor, you can see the monitor showing the camera feed from a few moments ago, which in turn shows the camera feed from a few moments even further back. This produces a strange "infinity mirror" effect, where you can see a sequence of nested images, each slightly delayed in time compared to the next one up.

Consider a simple setup of a camera pointed at a monitor, with a ball oscillating back and forth at frequency ω placed between them.

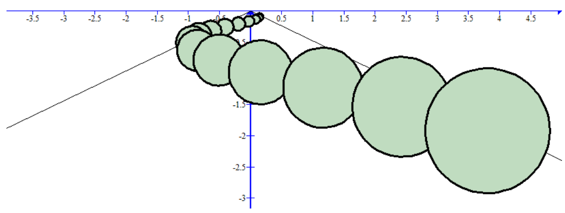
$$x_0 = A\sin(\omega t)$$



Each iterated image is delayed by total delay time Δt, and shrunk by some scale factor r:

$$\begin{aligned} x_i(t) &= r \cdot x_{i-1}(t - \Delta t) \\ &= r^i \sin\big(\omega(t - i \cdot \Delta t)\big) \end{aligned}$$

This creates appearance of a winding snake-like pattern, disappearing into the distance:



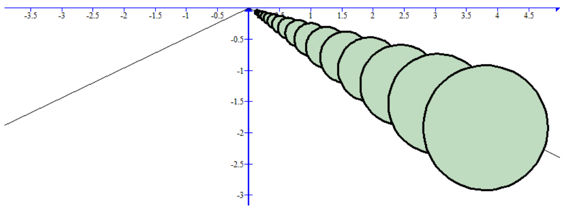
Un-Tuned Relay

MEASURING THE DELAY

The discrete nature of the images makes an interesting pattern possible. Suppose we tune the oscillation frequency so that its period is the same as the delay time, ie.:

$$\Delta t\omega = n \cdot 2\pi$$

In this way, each image is delayed by one full cycle, making it appear as though there is no delay at all:



Tuned Relay

$$
\begin{aligned}
x_i &= r^i \sin(\omega t - n \cdot 2\pi) \\
&= r^i \sin(\omega t)
\end{aligned}
$$

By tuning the oscillations, we can then get an estimate of the "resonant" frequency, and by extension the delay timescale.

Issues with Measurement

So we have a practical means of measuring the delay timescale, but we pretty quickly run into an issue when trying to measure the speed of light. The delay is a combination of two factors: the delay due to light travel (Δt_c), and the delay due to signal processing within the camera and monitor (Δt_p):

$$\Delta t = \Delta t_c + \Delta t_p$$

There is a complete overlap between the effects of these delays, and so we need Δt_c to be reasonably large compared to Δt_p. To be generous, let's say we can detect changes in the delay at 1 order of magnitude difference:

$$\frac{\Delta t_c}{\Delta t_p} \approx 0.1$$

In an actual camera setup, the processing speed will be dominated by the refresh rate of the monitor. At consumer grade estimates, this max out in the low 100's of Hz, meaning:

$$\Delta t_p \approx 10^{-2}$$

This requires:

$$\Delta t_c \approx 10^{-3}$$

We call this the **characteristic measurement time**, or "CMT". In this case, it corresponds to a delay distance of:

$$
\begin{aligned}
L &= \Delta t_c \cdot c \\
&= 10^{-3} \cdot 3 \cdot 10^{8} \\
&= 3 \cdot 10^{5}\, m \\
&= 30\, km
\end{aligned}
$$

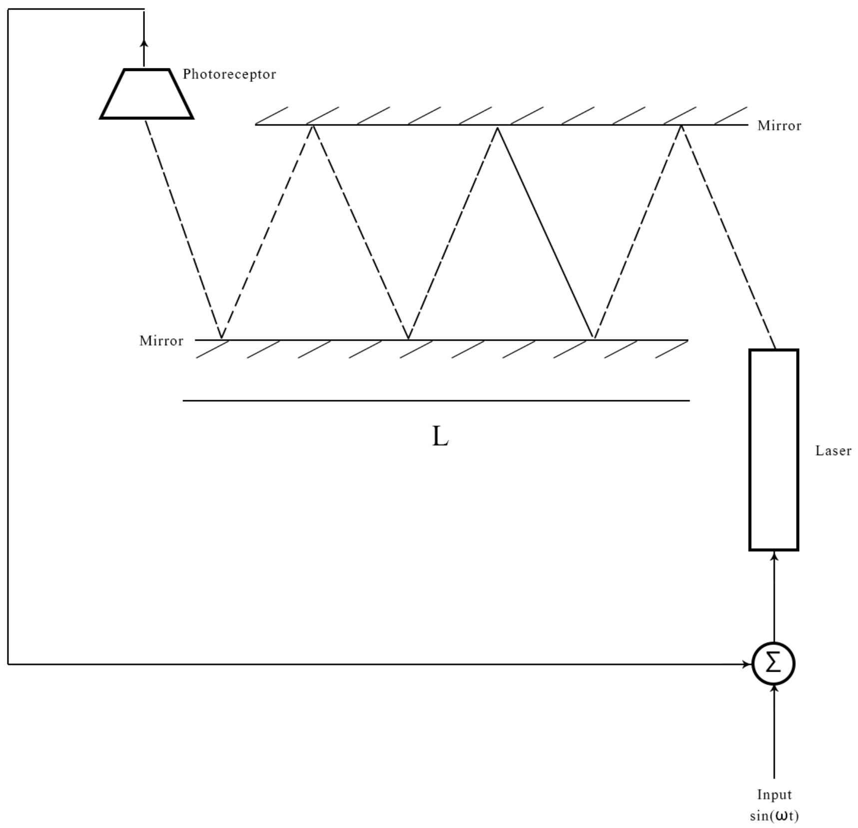
So, to measure this with any level of confidence, we would need to resolve a clear image over *at least* 30 km. In fact, we would need to clearly see past several times this distance, as identifying resonance requires us to see several iterated images. Though this may, strictly speaking, be possible, it is well beyond any practical limits.

ATTACKING WITH LASERS

So, the problem, as presented, is intractable. We can make things a little more reasonable if we get a bit abstract with our ideas of what a “camera” and “monitor” are:

Monitor	A device that takes an electrical signal input and outputs a corresponding light signal
Camera	A device that takes a light signal input and outputs a corresponding electrical signal

In the strictest sense, we could consider a simple photoreceptor to be a “camera”, and a laser to be a “monitor”. We can instead create a feedback loop between a laser (the monitor) and a photoreceptor that it is pointed at (a camera), with the laser also having some initial driving input signal (the oscillator). In this way, we can set up something like this:



We can also achieve long travel distances by bouncing the laser between mirrors, adjusting the reflection angle as needed to change the number of reflections/light travel time: Even 1 dozen reflections increases our measurement distance by an order of magnitude. We also side-step the monitor refresh rate issue by keeping things as close to “analogue” as possible.

Towards modelling: suppose we have some input signal (an AC current):

$$\psi_0(t) = A\cos(\omega_{mod} t)$$

Which is passed into the receptor, then fed back to the emitter with delay Δt, and damping factor λ:

$$\psi_i(t) = \psi_{i-1}(t - \Delta t) \cdot \lambda$$

Which is passed to the receptor, then fed back to the emitter with delay and damping factor :

$$\psi_1(t) = \psi_0(t - \Delta t) \cdot \lambda$$

This process obviously repeats for each iteration, such that the N'th relay is:

$$\psi_N(t) = \psi_N(t - N \ \cdot \Delta t) \cdot \lambda^N$$

Such that the total signal is the summation of these is:

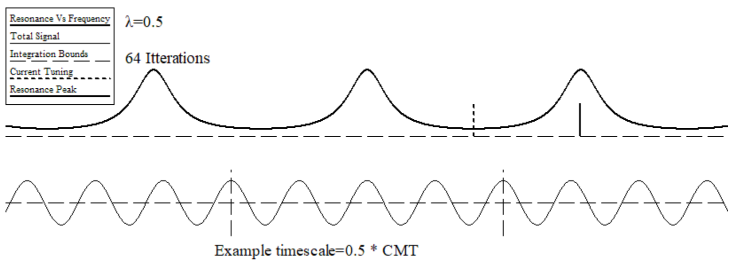
$$\psi(t) = A \sum_{N=0}^{\infty} \cos\left(\omega_{mod} \cdot (t - N\Delta t)\right) \cdot \lambda^N$$

Our major interest is to measure the intensity of this signal:

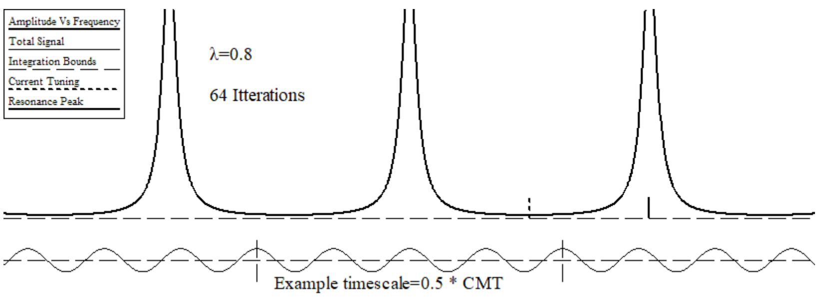
$$\langle \psi^2(t) \rangle$$

Which reaches a maximum when the input frequency resonates with the delay time, i.e.

$$\omega_{mod} = N\frac{2\pi}{\Delta t}:$$



High Damping Example



Low Damping Example

By trialling different input frequencies, we can locate these peaks, thereby measuring the value of Δt to a high degree of precision.

SOME ALTERNATIVES

Delay Times

If resonance doesn't work, we could also put in a flat signal, turn it off, and measure the rate at which the signal strength decays as the images fade.

$$\psi_0(t) = A \cdot u(-t)$$

The decay is a function of both R and Δt, and so will be less straightforward to analyse

Altering delay timescale

If it's not an option to vary the input frequency, we can alternately intentionally induce a phase shift in the electrical transmission from the photoreceptor to the laser. In this way, we can tune Δt to match ω, instead of the other way around.